Classifying Signals on Irregular Domains via Convolutional Cluster Pooling

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1. Motivations

We are surrounded by data lying on an underlying non-euclidean structure (e.g. 3D skeleton data, social networks and chemical compounds). Graph Convolutional Networks (GCNs) [1, 2, 3] provide a comprehensive and solid framework for vertex classification in such data domains, because they model directly the topological structures through edge weights. Differently, we focus on graph signal classification.

Our approach, built by stacking multiple Convolutional Cluster Pooling (CCP) layers, provides:

- a hierarchical framework for supervised learning in homogeneous graph contexts.
- a spatial formulation for graph filtering which, as for CNNs, exploits weight sharing and produces a pooled graph signal.

2. Our Proposal

Multiple applications of the CCP layer lead to multi-scale representations of the graph.

- Firstly, CCP performs a clustering operation on the input graph, resulting in a coarser output graph, whose affinity matrix reflects relationships among clusters regressed at training time.
- Secondly, the layer selects for each cluster a fixed number of candidate nodes for the aggregation phase, and sorts them depending on a centrality-based rank within the cluster.

3. Hierarchical Soft Clustering

A cascade of M soft-partitions, described by an ordered sequence of assignment matrices \( K^{(1)}, K^{(2)}, \ldots, K^{(M)} \), forms a soft dendrogram for the original graph \( A \). The problem of obtaining a good dendrogram is formalised as follows:

\[
\max_{K^{(1)} \ldots K^{(M)}} \mathcal{L}_{K} = \frac{1}{2} \sum_{m=1}^{M} \sum_{i=1}^{\left|\mathcal{K}^{(m)}\right|} \text{Cohesion}(K(i)^{(m)}) \text{Vol}(K(i)^{(m)})
\]

subject to

\[
\sum_{k=1}^{\left|\mathcal{K}^{(m)}\right|} K(i, k)^{(m)} = 1 \quad i = 1,2,\ldots,\left|\mathcal{K}^{(m-1)}\right|, \quad m = 1,2,\ldots,M.
\]

This way, we require intermediate clusters with maximal cohesion and minimum size. \( \mathcal{L}_{K} \) is paired to the classification loss \( \mathcal{L}_{c} \), delivering the regularised objective \( \mathcal{L} = \mathcal{L}_{c} + \mathcal{L}_{K} \). This way, the supervision signal may provide information to the process of clusters formation.

4. Neighbourhood Selection

For each cluster we select as candidate set \( V_{i}^{(m+1)} \) for the filtering stage the set containing the most \( L \) representative nodes.

We consider a node more central if: i) it has a high membership value for the cluster under consideration; ii) it is a large part of its direct neighbours nodes share the same cluster in the graph.

5. Convolutional Cluster Pooling (CCP)

The \( m \)-th CCP layer takes in input an affinity matrix \( A^{K_{m}} \) and a multi-dimensional \( \mathcal{T}^{(m)} \in \mathbb{R}^{d_{X} \times d_{L}} \) signal defined on the vertex set.

**CCP yields:**

- \( A^{K_{m+1}} \): a new reduced affinity matrix
- \( \mathcal{T}^{(m+1)} \in \mathbb{R}^{d_{X+1} \times d_{L}} \): a pooled signal

where \( W \in \mathbb{R}^{d_{X} \times d_{K}} \) and \( b \in \mathbb{R}^{d_{R}} \) are parameters of our CCP layer.

6. GCN Baselines

Comparison w.r.t other graph censering and filtering approaches.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Coarse</th>
<th>CIFAR-10</th>
<th>NTU-CS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chebyshev [1]</td>
<td>Gracus</td>
<td>78.15</td>
<td>74.85</td>
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<tr>
<td>GCN [2]</td>
<td>Gracus</td>
<td>67.01</td>
<td>62.00</td>
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<tr>
<td>GAT [3]</td>
<td>Gracus</td>
<td>72.82</td>
<td>59.48</td>
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<tr>
<td>CCP</td>
<td>CCP</td>
<td>84.4</td>
<td>80.1</td>
</tr>
</tbody>
</table>

7. Classification Results

<table>
<thead>
<tr>
<th>Action recognition</th>
<th>NTU RGB+D</th>
<th>CIFAR-10</th>
<th>Text categorisation</th>
<th>20NEWS</th>
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</thead>
<tbody>
<tr>
<td>P-LSTM</td>
<td>62.9</td>
<td>Graph-CNNs</td>
<td>68.3</td>
<td>Linear SVM</td>
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<td>TGCNN</td>
<td>71.4</td>
<td>FC</td>
<td>78.6</td>
<td>FC2500-FCS00</td>
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<tr>
<td>Deep STGCN</td>
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<td>CCP</td>
<td>84.4</td>
<td>Softmax</td>
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<tr>
<td>C-CNN</td>
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<td>Stochastic Pooling</td>
<td>84.9</td>
<td>Chebyshev - GC32</td>
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<tr>
<td>CCP</td>
<td>80.1</td>
<td>ResNet</td>
<td>93.6</td>
<td>CCP</td>
</tr>
</tbody>
</table>

8. Learned Receptive Fields (CIFAR-10)

1st CCP  2nd CCP  3rd CCP  4th CCP

9. References